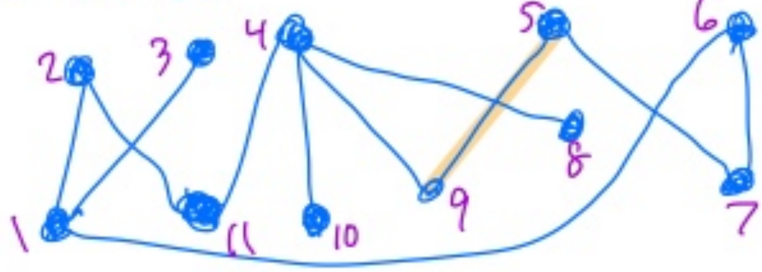
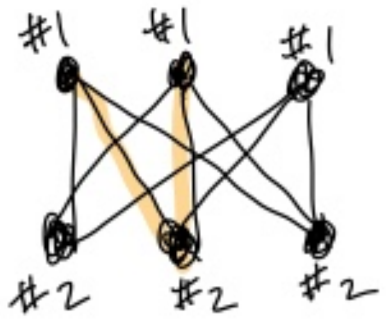
Questions

① What is the chromatic # of $K_{3,3}$?

② Is this a tree?



①



Chromatic # is 2 — let one color be used for the first set of vertices, another for the second set.
(The chromatic # can't be 1, because edges connect vertices, so there must be at least 2 colors).

We have shown $K_{3,3}$'s chromatic # is 2.

The same proof works to show all bipartite graphs have chromatic # 2.

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② No. $5 \to 9 \to 4 \to 11 \to 2 \to 1 \to 6 \to 7 \to 5$ is a circuit in the graph, so this is not a tree.

Tips from homework & test:

- Be careful about implications and hypotheses on theorems.

- Dirac graph thm: If a graph is simple, connected, has n vertices with $n \geq 3$ and each vertex has at least degree $\frac{n}{2}$, then the graph contains a Hamiltonian circuit.

Not every Hamiltonian circuit satisfies this.

- Euler Characteristic Formula

$$\chi = v - e + f$$

2 for planar or spherical graph.

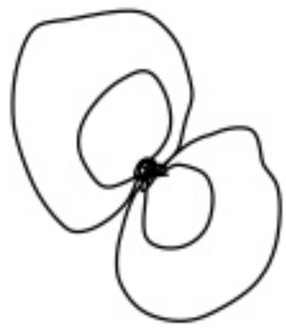
Always true for connected graphs — don't have to be simple — graphs have to be embedded — edges don't cross.

- For a simple connected planar graph, $e \leq 3v - 6$.

(we used the Euler characteristic to prove it).

Not all planar graphs satisfy this, because they're not all simple.

Silly example



$v = 1$

$e = 4$

$f = 5$

$v - e + f = 1 - 4 + 5$
$= 2$

$\underset{4}{e} \not\leq \underbrace{3v - 6}_{-3}$

From last time

Simple connected graph with no circuits.

Fact Given a tree with $V = n$ vertices, there are $(n-1)$ edges.

Proof: We will prove by induction on n.

Base case: Suppose $n=1$. Then the tree has only 1 vertex and no edges. Thus, the theorem holds in this case.

Next, assume we know the theorem to be true if the number of vertices is between 1 & k, where $k \geq 1$ is a positive integer. induction hypothesis.

Now suppose we have a tree with $k+1$ vertices. If we delete one vertex that is on a leaf of the tree, including its connecting edge, we still have a tree, but with k vertices. By the induction hypothesis, the new tree with k vertices has $k-1$ edges.

$\therefore$ Since we have just removed one edge, our original ~~graph~~ tree with $k+1$ vertices has

$$k = k-1+1 \text{ edges.}$$

Therefore, the theorem is also true for a tree with $k+1$ vertices.

By induction, the theorem is true for any tree with n vertices, where n is <u>any</u> positive integer. $\boxed{\not\!\!/}$

More refined information for k-ary trees.

($k=2$ binary ) $k=3$ 

<u>Fact</u>: For a full k-ary tree j internal vertices, the whole tree has $n = kj+1$ vertices

eg 

(internal vertex: degree is >1)

7 internal vertices

$k=2$

$n = 2\cdot 7+1 = 15$